

Rossmoyne Senior High School

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3 Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **ten (10)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(7 marks)

(a) The first three terms, in order, of a geometric sequence are 1400, 980 and 686.

(i) Deduce a rule for the n^{th} term of this sequence. (2 marks)

Solution
$r = 980 \div 1400 = 0.7$ $T_n = 1400(0.7)^{n-1}$
Specific behaviours
✓ calculates ratio ✓ writes the rule

(ii) Calculate the 4^{th} term of the sequence. (1 mark)

Solution
$T_4 = 480.2$
Specific behaviours
✓ writes exact value of term

(b) The first three terms, in order, of an arithmetic sequence are 2.4, 5.5 and 8.6.

(i) The rule for the n^{th} term of this sequence can be written in the form $T_n = pn + q$. Determine the values of p and q . (3 marks)

Solution
$d = 5.5 - 2.4 = 3.1$ $T_n = 2.4 + (n - 1)(3.1)$ $= 3.1n - 0.7$ $p = 3.1, \quad q = -0.7$
Specific behaviours
✓ calculates difference ✓ writes the rule and simplifies into required form ✓ states values

(ii) Calculate the 177^{th} term of the sequence. (1 mark)

Solution
$T_{177} = 548$
Specific behaviours
✓ writes exact value of term

Question 10

(10 marks)

A public relations company was tasked with determining whether a person's support for a sugary drinks tax could be associated with their interest in the news.

The company carried out a telephone survey, where people could respond to two questions as shown in the following table:

Question	Choice of response
Are you interested in the news?	Yes or No
Do you support a sugary drinks tax?	Yes, No or Undecided

The responses to the telephone survey are summarised in this table:

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	528	374	221
	No	196	204	146

(a) Calculate the number of people who

(i) answered no to being interested in the news. (1 mark)

Solution
$196 + 204 + 146 = 546$
Specific behaviours
✓ adds to get number

(ii) responded to the survey. (1 mark)

Solution
$1123 + 546 = 1\ 669$
Specific behaviours
✓ adds to get number

(b) If there was an association between interest in the news and support for a sugary drinks tax, should the company expect the same percentage of those who support a sugary drinks tax to be interested in the news compared to those who do not support a sugary drinks tax? Explain your answer. (2 marks)

Solution
No. If there was an association then the percentages would be different.
Specific behaviours
✓ indicates no
✓ explanation indicating different percentages

- (c) Complete the two-way table below to show the associated **row** percentages for the previous table, rounding percentages to the nearest whole number. (3 marks)

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	47%	33%	20%
	No	36%	37%	27%

Solution
See table
Specific behaviours
✓ first row, ✓ second row, ✓ rounds correctly

- (d) What percentage of those who are interested in the news support a sugary drinks tax? (1 mark)

Solution
47%
Specific behaviours
✓ percentage from table

- (e) In the context of the task they were given, how should the public relations company interpret the responses to their survey? (2 marks)

Solution
The PR company should conclude that an association exists between interest in the news and support for a sugary drinks tax. For example, 47% of those interested in the news supported a sugary drinks tax compared to only 36% of those who were not interested in the news.
Specific behaviours
✓ identifies an association exists ✓ justifies with reference to different percentages in at least one column

Question 11

(9 marks)

To save money towards a deposit on a house, Anh and Bo started an investment account. They made an initial deposit of \$3 700, and then deposited an extra \$850 at the end of each month for the next year. Interest on the account was to be calculated and paid monthly.

The table below shows the progress of their savings for the first few months.

Month (n)	Balance of account at start of month (T_n)	Interest added at end of month	Deposit made at end of month	Balance of account at end of month (T_{n+1})
1	\$3 700.00	\$14.80	\$850.00	\$4 564.80
2	\$4 564.80	\$18.26	\$850.00	\$5 433.06
3	\$5 433.06	\$21.73	\$850.00	\$6 304.79
4	\$6 304.79	A	\$850.00	B

- (a) What was the monthly percentage interest rate? (1 mark)

Solution
$14.80 \div 3700 \times 100 = 0.4\%$ (accept 0.004)
✓ calculates decimal or percentage

- (b) Determine the values of **A** and **B** in the table. (2 marks)

Solution
$A = 6304.79 \times 0.004 = \25.22
$B = 6304.79 + 25.22 + 850 = \7180.01
✓ Calculates A correctly ✓ Calculates B

- (c) Write a recursive rule to determine the balance of the account at the start of each month. (2 marks)

Solution
$T_{n+1} = 1.004 T_n + 850, \quad T_0 = 3700$
Specific behaviours
✓ Writes the rule ✓ Writes first term

- (d) Determine, to the nearest dollar, the balance of the account at the start of month 13. (2 marks)

Solution
$T_{13} = \$14\,308.98$ $\approx \$14\,309$
Specific behaviours
✓ Writes the balance ✓ Rounds correctly

- (e) Calculate, to the nearest dollar, the total interest earned up to the start of month 13. (2 marks)

Solution
$3700 + (12 \times 850) = \$13\,900$ $14\,309 - 13\,900 = \$409$
Specific behaviours
✓ Calculates the total deposited ✓ Deducts from answer in part (d) to get interest

Question 12

(7 marks)

An employer is considering two pay schemes for new employees:

Scheme A

For the first month, a new employee is paid \$750. In subsequent months, their pay is increased so that it is \$40 more than the previous month.

Scheme B

For the first month, a new employee is paid \$750. In subsequent months, their pay is increased so that it is 5% more than the previous month.

- (a) Write a recursive rule for A_n , the payment made in month n to an employee using Scheme A and another recursive rule for B_n , the payment made in month n to an employee using Scheme B. (3 marks)

Solution
$A_{n+1} = A_n + 40 \quad A_1 = 750$ $B_{n+1} = B_n \times 1.05 \quad B_1 = 750$
<ul style="list-style-type: none"> ✓ Writes the A_n rule ✓ Writes the B_n rule ✓ Writes the first terms

- (b) In which month does the payment using Scheme B first exceed that using Scheme A, and by how much? (2 marks)

Solution
Month 5: $A_5 = 910.00$ $B_5 = 911.63$ $911.63 - 910.00 = \$1.63$
<ul style="list-style-type: none"> ✓ Indicates month 5 ✓ Determines how much Scheme B exceeds Scheme A

- (c) If an employee only worked for 6 months, which pay scheme should they choose in order to gain the most pay? Justify your answer. (2 marks)

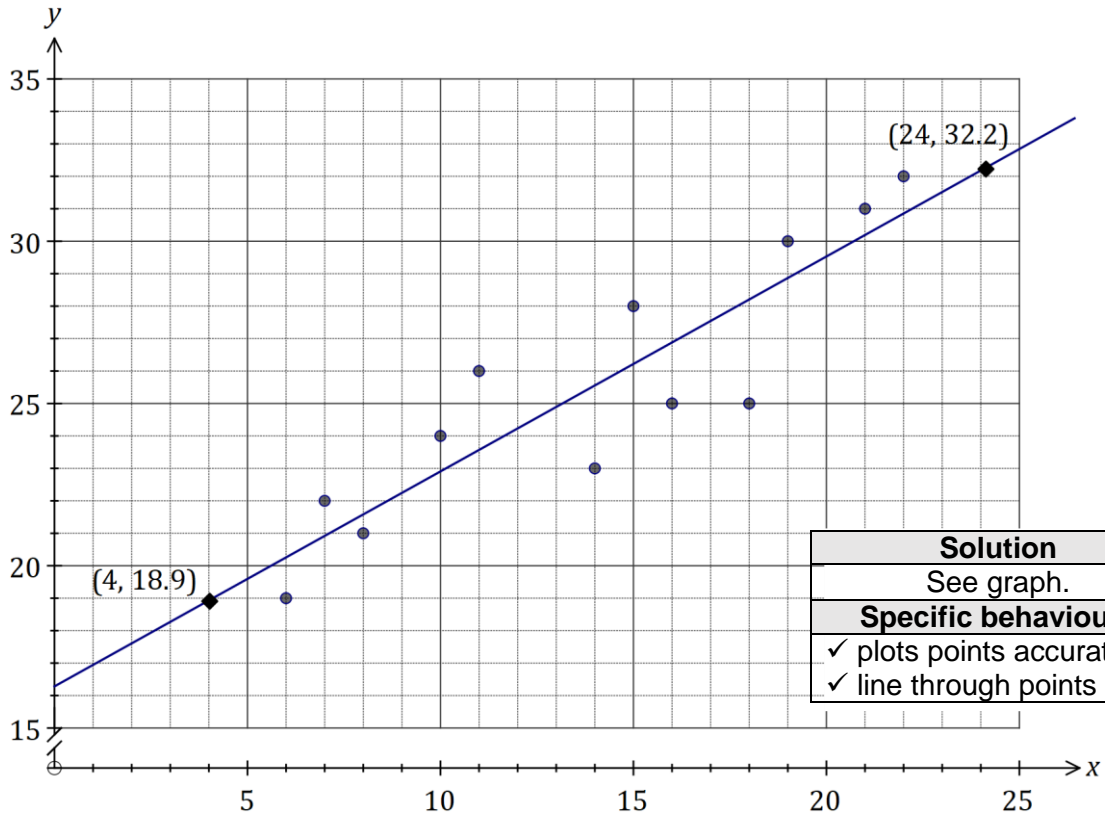
Solution
Scheme B Sum of first months for: Scheme A: \$5100.00 Scheme B: \$5101.43 (\$1.43 more than A)
<ul style="list-style-type: none"> ✓ Chooses Scheme B ✓ Shows 6 month values of both schemes or difference

Question 13

(15 marks)

A tomato grower added varying amounts of a liquid fertiliser (x ml) to the irrigation systems of twelve greenhouses and observed the resulting yield of tomatoes per plant (y kg). A sample of the data recorded is shown in the table and scatterplot below.

x (mL)	6	7	8	10	11	14	15	16	18	19	21	22
y (kg)	19	22	21	24	26	23	28	25	25	30	31	32



Solution
See graph.
Specific behaviours
✓ plots points accurately
✓ line through points

(a) Name the response variable.

Solution
y (yield per plant)
Specific behaviours
✓ correct variable

(1 mark)

(b) For this data, calculate

(i) the correlation coefficient.

Solution
$r = 0.893$
Specific behaviours
✓ value of r

(1 mark)

(ii) the values of a and b in the equation of the least-squares line $y = ax + b$.

Solution
$a = 0.662, \quad b = 16.28$
Specific behaviours
✓ gradient (a)
✓ y -intercept (b)

(2 marks)

- (c) What percentage of the variation in the yield per plant can be explained by the variation in the amount of liquid fertiliser added? (1 mark)

Solution
$r^2 = 0.798$ which is 79.8%.
Specific behaviours
✓ states correct percentage

- (d) If the amount of liquid fertiliser added to the irrigation system in a greenhouse was increased by one millilitre, what increase in the yield of tomatoes per plant can be expected? Explain your answer. (2 marks)

Solution
An increase of 0.662 kg per plant. This is the value of the gradient of the least-squares line.
Specific behaviours
✓ value ✓ indicates use of gradient

- (e) If no liquid fertiliser was used, what yield of tomatoes per plant does the linear model predict? (1 mark)

Solution
A yield of 16.3 kg.
Specific behaviours
✓ uses value of y -intercept in kg

- (f) Use the equation of the least-squares line to calculate the value of y when $x = 4$ and when $x = 24$. (2 marks)

Solution
$x = 4, \quad y = 18.9$
$x = 24, \quad y = 32.2$
Specific behaviours
✓ one correct value ✓ both correct values

- (g) Use your answers to part (f) to draw the least-squares line on the scatterplot. (2 marks)

- (h) Estimate the yield of tomatoes per plant when 13 ml of liquid fertiliser is added to the irrigation system and comment on the reliability of this value. (3 marks)

Solution
$x = 13 \text{ ml}, \quad y = 24.9 \text{ kg per plant}$
Estimate is reliable as correlation is strong and it involves interpolation.
Specific behaviours
✓ any value that rounds to 25 kg ✓ considers reliability ✓ considers interpolation

Question 14

(10 marks)

A painting was sold for \$30 000 by an art gallery.

- (a) The painting could be bought on terms in which a premium of 7.5% was added to the purchase price and then the total amount repaid to the gallery in 24 equal monthly repayments. How much would each repayment be? (2 marks)

Solution
$30000 \times 1.075 = \$ 32250$
$32250 \div 24 = \$ 1343.75$ per month
Specific behaviours
<ul style="list-style-type: none"> ✓ determines price with premium ✓ determines value of repayment

- (b) The painting was expected to increase in value at an annual rate of 9%. What is the expected value of the painting 4 years after it was purchased, to the nearest one hundred dollars? (2 marks)

Solution
$30000 \times 1.09^4 = \$ 42347.45$ $\approx \$ 42300$
Specific behaviours
<ul style="list-style-type: none"> ✓ Multiplies value of painting by 1.09^4 ✓ Rounds to nearest hundred dollars

- (c) An amount of \$30000 can so be invested using two different saving schemes:
Scheme A, in which interest of 6.21% per annum is compounded monthly; and
Scheme B, in which interest of 6.29% per annum is compounded semi-annually.
- (i) Calculate the effective interest rates of both schemes, correct to 4 decimal places, and hence state which scheme would pay the most interest over three years. (3 marks)

Solution	
$E_A = \left(1 + \frac{0.0621}{12}\right)^{12} - 1 = 0.063898369$	$E_B = \left(1 + \frac{0.0629}{2}\right)^2 - 1 = 0.063889103$
Rate is 6.3898% pa (4dp)	Rate is 6.3889% pa (4dp)
Scheme A would pay the most interest	
Specific behaviours	
<ul style="list-style-type: none"> ✓ determines effective rate for A and B (accept values in decimals) ✓ writes both to 4dp ✓ indicates Scheme A as paying most interest over 3 years 	

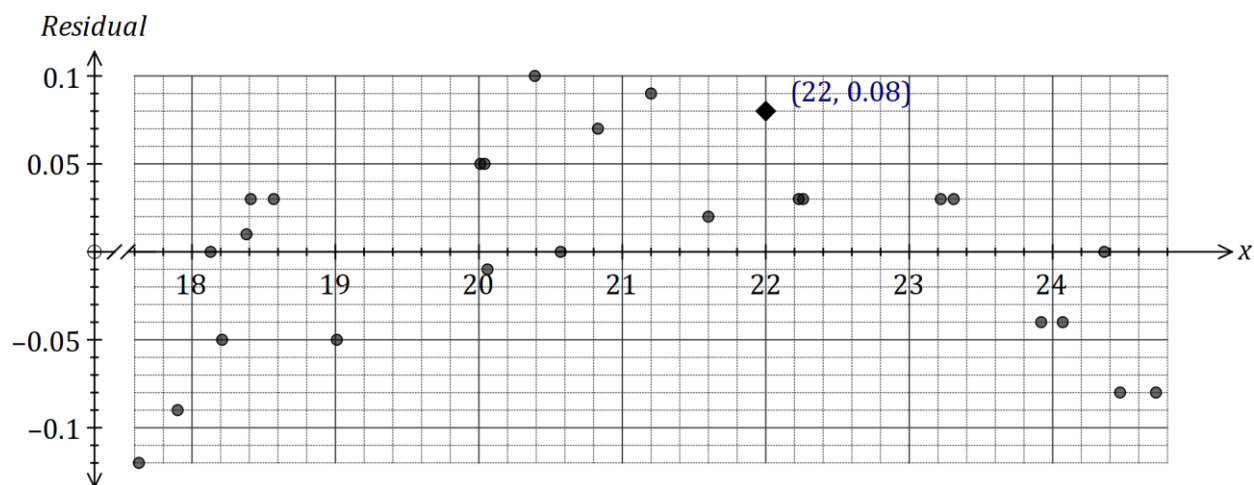
- (ii) Calculate the interest that accrues over three years on the \$30000 using the scheme which pays the most interest. (3 marks)

Solution																	
<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th colspan="2" style="text-align: left; font-size: small;">Compound Interest</th> </tr> </thead> <tbody> <tr><td>N</td><td>3</td></tr> <tr><td>I%</td><td>6.21</td></tr> <tr><td>PV</td><td>30000</td></tr> <tr><td>PMT</td><td>0</td></tr> <tr><td>FV</td><td>-36126.15026</td></tr> <tr><td>P/Y</td><td>1</td></tr> <tr><td>C/Y</td><td>12</td></tr> </tbody> </table>	Compound Interest		N	3	I%	6.21	PV	30000	PMT	0	FV	-36126.15026	P/Y	1	C/Y	12	$30000 \times \left(1 + \frac{0.0621}{12}\right)^{12 \times 3} = \$ 36126.15$ $36126.15 - 30000 = \$ 6126.15$ interest
Compound Interest																	
N	3																
I%	6.21																
PV	30000																
PMT	0																
FV	-36126.15026																
P/Y	1																
C/Y	12																
Specific behaviours																	
<ul style="list-style-type: none"> ✓ compounds for 3 years using answer in c (i) ✓ determines value after 3 years ✓ deducts original price and determines interest accrued 																	

Question 15

(8 marks)

A linear model was fitted to a set of data, resulting in a correlation coefficient of $r = 0.94$ and a least-squares line with equation $\hat{y} = 5.16 + 0.16x$. A residual plot for the linear model is shown below.



(a) Calculate, and add to the plot above, the residual for the point $x = 22, y = 8.76$.

(3 marks)

Solution
$\hat{y} = 5.16 + 0.16(22) = 8.68$
Residual = $8.76 - 8.68 = 0.08$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates \hat{y} ✓ calculates residual ✓ plots residual

(b) Use the residual plot to comment on the appropriateness of fitting a linear model to the data.

(2 marks)

Solution
A pattern is evident in the residual plot (as x values increase, residuals start mostly negative, then positive and then negative again) and so a linear model is not appropriate.
Specific behaviours
<ul style="list-style-type: none"> ✓ not appropriate ✓ reason

(c) Determine the y -coordinate of the point with a residual of 0.07 on the above plot.

(3 marks)

Solution
$x = 20.8$
$\hat{y} = 5.16 + 0.16(20.8) = 8.488$
$y - 8.488 = 0.07 \Rightarrow y = 8.558$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates x-coordinate ✓ calculates \hat{y} ✓ calculates y

Question 16

(13 marks)

When Atarcoin, a new cryptocurrency was launched, one Atarcoin was valued at \$4.00. After one week of trading, the value of Atarcoin had increased to \$5.00, and after another week had increased to \$6.25.

- (a) Show that the value of Atarcoin increased by 25% each week. (2 marks)

Solution
$5 \div 4 = 1.25,$ $6.25 \div 5 = 1.25 \Rightarrow 25\% \text{ increase}$ or $4 \times 1.25 = 5,$ $5 \times 1.25 = 6.25$
Specific behaviours
✓ shows one ratio (or multiple) ✓ shows both ratios (or multiples)

The value of Atarcoin, V_n in dollars, n weeks after its launch date, can be modelled by the recurrence relation $V_{n+1} = 1.25V_n$, $V_0 = 4$.

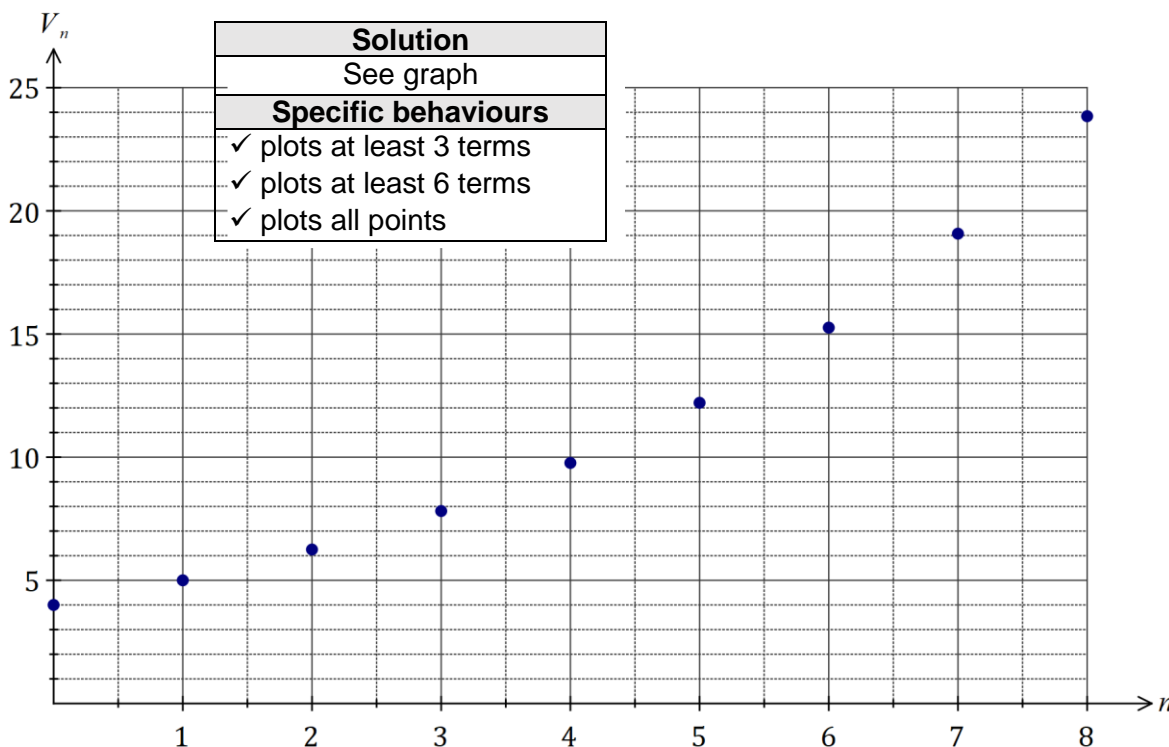
- (b) Calculate the value of Atarcoin ten weeks after its launch date. (1 mark)

Solution
$V_{10} = \$37.25$
Specific behaviours
✓ value

- (c) At the end of which week did the value of Atarcoin first exceed \$100? (1 mark)

Solution
End of week 15.
Specific behaviours
✓ week

- (d) Graph V_n against n on the axes below. (3 marks)



The value of Atarcoin peaked at the end of week 38, and from that time onwards, its value fell by 30% each week.

- (e) Determine the value of Atarcoin at the end of week 39. (2 marks)

Solution
$V_{38} = 19259.30$
$T_{39} = V_{38} \times 0.7 = \$13\,481.51$
Specific behaviours
<ul style="list-style-type: none"> ✓ peak value ✓ value one week later

- (f) Ignoring any fees involved in buying and selling a cryptocurrency, determine the profit or loss made by a person who bought 25 Atarcoins eight weeks after their launch, held them for 40 weeks and then sold them all. (4 marks)

Solution
$V_8 = 23.84$
$38 - 8 = 30, 40 - 30 = 10$ Sells 10 weeks after peak
$T_{n+1} = 0.7T_n, \quad T_0 = 19259.30$ $T_{10} = 544.03$
Profit = $25 \times (544.03 - 23.84)$ $= 25 \times 520.19$ $= \$13\,004.75$
Specific behaviours
<ul style="list-style-type: none"> ✓ value when bought ✓ week sold ✓ value when sold ✓ profit

Question 17

(9 marks)

The temperature, °C, of an industrial oven n minutes after it is turned on can be modelled by

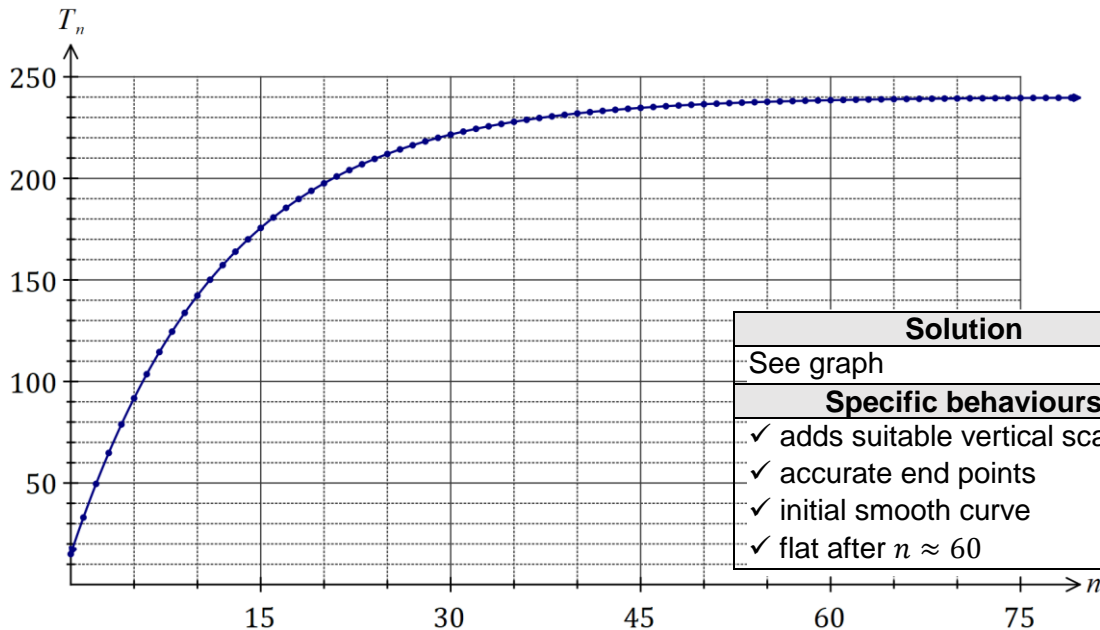
$$T_{n+1} = 0.92T_n + 19.2, \quad T_0 = 15$$

- (a) Use the recurrence relation to complete the table of values below, rounding the temperature to the nearest °C. (2 marks)

n	0	10	20	30	40
T_n	15	142	198	222	232

Solution
See table
Specific behaviours
✓ at least 3 values correct, ✓ all values correct

- (b) Sketch a graph of the temperature of the oven for the first 75 minutes on the axes below. Make sure you add a suitable scale to the vertical axis. (4 marks)



Solution
See graph
Specific behaviours
✓ adds suitable vertical scale ✓ accurate end points ✓ initial smooth curve ✓ flat after $n \approx 60$

- (c) The manufacturer claims that the oven will reach within 10 °C of its maximum temperature within 30 minutes of being turned on. Comment on this claim. (3 marks)

Solution
Maximum temperature is 240°C. Temperature exceeds 230°C after 38 minutes or at 30mins temperature is 222°C which is not within 10°C of maximum. Claim is false.
Specific behaviours
✓ indicates maximum temperature ✓ states time reaches within 10°C or is not within at 30mins ✓ comment on claim

Question 18

(10 marks)

An annuity paying a monthly sum of \$2 500 is set up with an initial sum of \$500 000 and interest of 6.2% per annum compounded monthly.

- (a) The balance of the loan at the start of month n is given by the recurrence relation $A_{n+1} = rA_n - d$, $A_1 = 500000$. State the values of r and d . (2 marks)

Solution
$0.062 \div 12 = 0.005167 \rightarrow r = 1.0052$
$d = 2500$
Specific behaviours
<ul style="list-style-type: none"> ✓ determines r ✓ determines d

- (b) Determine the value of the annuity after twelve months and comment on what this figure indicates. (3 Marks) (2 marks)

Solution
\$ 501 028.91
The value has increased, indicating that the annuity can pay out \$2 500 forever and the value of the annuity will also increase forever
Specific behaviours
<ul style="list-style-type: none"> ✓ Determines the value of the annuity ✓ Indicates the annuity is increasing ✓ hence withdrawing \$2500 never reduces the initial sum below \$500 000

Compound Interest	
N	12
I%	6.2
PV	-500000
PMT	2500
FV	501028.9118
P/Y	12
C/Y	12

- (c) Determine to the nearest dollar, the monthly sum that should be withdrawn from the annuity if

- (i) The annuity is to last for 20 years.

Solution
\$ 3640
Specific behaviours
<ul style="list-style-type: none"> ✓ monthly amount to the nearest dollar

Compound Interest	
N	240
I%	6.2
PV	-500000
PMT	3640.08446
FV	0
P/Y	12
C/Y	12

- (ii) The annuity is to be a perpetuity.

Solution
$500000 \times (0.052 \div 12) = \$ 2152.92$
Specific behaviours
<ul style="list-style-type: none"> ✓ monthly sum to become perpetual

- (d) If the interest rate of 6.2% was halved after one year, calculate the total interest accrued by the annuity over the first two years. (3 marks)

Solution
During first year: $501028.91 - 500000 + 12 \times 2500 = \$ 31\ 028.91$
During second year: $486353.46 - 501028.91 + 12 \times 2500 = \$ 15\ 324.55$
Total: $31\ 028.91 + 15\ 324.55 = \$ 46\ 353.46$
Specific behaviours
<ul style="list-style-type: none"> ✓ determines the interest in first year ✓ determines the interest in second year ✓ adds the interests together

Compound Interest	
N	12
I%	3.1
PV	-501028.9118
PMT	2500
FV	486353.4599
P/Y	12
C/Y	12

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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